

Due Wednesday, February 26, 2025.

Write your homework *neatly, in pencil*, on blank white $8\frac{1}{2} \times 11$ printer paper. Always *write the problem*, or at least enough of it so that your work is readable. If the problem involves a function, write the function. If the problem involves an equation, write the equation. Use words, and when appropriate, *write in sentences*.

Definition 1. Define the *natural logarithm* to be the function

$$\log : (0, \infty) \rightarrow \mathbb{R} \quad \text{given by} \quad \log(x) = \int_1^x \frac{1}{t} dt.$$

By the Fundamental Theorem of Calculus (FTC) and the Mean Value Theorem (MVT Corollary 2), we have

$$\frac{d}{dx} \log(x) = \frac{1}{x} \quad \text{and} \quad \int \frac{1}{x} dx = \log(x) + C.$$

Engineers write $\ln(x) = \log(x)$.

Problem 1 (Thomas §7.2 # 13). Find $\frac{dy}{dx}$ where

$$y = \ln x^3.$$

Problem 2 (Thomas §7.2 # 21). Find $\frac{dy}{dx}$ where

$$y = \frac{\ln x}{1 + \ln x}.$$

Problem 3 (Thomas §7.2 # 45). Compute

$$\int_2^4 \frac{dx}{x(\ln x)^2}.$$

Problem 4 (Thomas §3.5 # 35). Find $\frac{dr}{d\theta}$ where

$$r = \sin(\theta^2) \cos(2\theta).$$

Problem 5 (Thomas §3.6 # 25). Find $\frac{dy}{dx}$ where

$$y^2 = \frac{x-1}{x+1}.$$

Problem 6 (Thomas §5.5 # 35). Compute

$$\int \frac{\sin(2t+1)}{\cos^2(2t+1)} dt.$$

Problem 7 (Thomas §3.5 # 60). Suppose that the functions f and g and their derivatives with respect to x have the following values at $x = 0$ and $x = 1$.

| x | $f(x)$ | $g(x)$ | $f'(x)$ | $g'(x)$ |
|-----|--------|--------|---------|---------|
| 0 | 1 | 1 | 5 | $1/3$ |
| 1 | 3 | -4 | $-1/3$ | $-8/3$ |

Find the derivatives with respect to x of the following combinations at the given value of x .

(a) $5f(x) - g(x)$, $x = 1$

(b) $f(x)g^3(x)$, $x = 0$

(c) $\frac{f(x)}{g(x) + 1}$, $x = 1$

(d) $f(g(x))$, $x = 0$

(e) $g(f(x))$, $x = 0$

(f) $(x^{11} + f(x))^{-2}$, $x = 1$

(g) $f(x + g(x))$, $x = 0$

Problem 8. Let

$$f(x) = 6x^3 - 11x^2 - 24x + 9.$$

Note that $f(3) = 0$. Find all zeros of f .

Problem 9. Consider the family of functions $f(x) = x^4 - ax^2$. Show that f has a local maximum if and only if f has 3 distinct zeros.

Problem 10 (Thomas Problem §4.5 # 12). Find the volume of the largest right circular cone that can be inscribed in a sphere of radius 3.

